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Geometric Transform

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Outline

- **Inverse Transformations**
- **Two-Dimensional Composite Transformations**
- **Composite Two-Dimensional Translations**
- **Composite Two-Dimensional Rotations**
- **Composite Two-Dimensional Scaling**
- **General Two-Dimensional Pivot-Point Rotation**

Inverse Transformations

- Inverse translation matrix
  - Translate in the opposite direction

$$T^{-1} = \begin{bmatrix} 1 & 0 & -t_x \\ 0 & 1 & -t_y \\ 0 & 0 & 1 \end{bmatrix}$$

- Inverse rotation matrix
  - Rotate in the clockwise direction

$$R^{-1} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- Inverse scaling matrix

$$S^{-1} = \begin{bmatrix} \frac{1}{s_x} & 0 & 0 \\ 0 & \frac{1}{s_y} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Two-Dimensional Composite Transformations

Forming products of transformation matrices is often referred to as a concatenation, or composition, of matrices

We do premultiply the column matrix by the matrices representing any transformation sequence

since many positions in a scene are typically transformed by the same sequence, it is more efficient to first multiply the transformation matrices to form a single composite matrix

$$\boxed{\begin{aligned} P' &= M_2 \cdot M_1 \cdot P \\ &= M \cdot P \end{aligned}}$$

Composite Two-Dimensional Translations

- If two successive translation vectors (t_{1x}, t_{1y}) and (t_{2x}, t_{2y}) are applied to a 2-D coordinate position P, the final transformed location P' is

$$\begin{aligned} \mathbf{P}' &= \mathbf{T}(t_{2x}, t_{2y}) \cdot \{\mathbf{T}(t_{1x}, t_{1y}) \cdot \mathbf{P}\} \\ &= \{\mathbf{T}(t_{2x}, t_{2y}) \cdot \mathbf{T}(t_{1x}, t_{1y})\} \cdot \mathbf{P} \end{aligned}$$

- The composite transformation matrix for this sequence of translations is

$$\begin{bmatrix} 1 & 0 & t_{2x} \\ 0 & 1 & t_{2y} \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & t_{1x} \\ 0 & 1 & t_{1y} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_{1x} + t_{2x} \\ 0 & 1 & t_{1y} + t_{2y} \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{T}(t_{2x}, t_{2y}) \cdot \mathbf{T}(t_{1x}, t_{1y}) = \mathbf{T}(t_{1x} + t_{2x}, t_{1y} + t_{2y})$$

Composite Two-Dimensional Rotations

- Two successive rotations applied to a point P

$$\begin{aligned} P' &= R(\theta_2) \cdot \{R(\theta_1) \cdot P\} \\ &= \{R(\theta_2) \cdot R(\theta_1)\} \cdot P \end{aligned}$$

- We can verify that two successive rotations are additive:

$$R(\theta_2) \cdot R(\theta_1) = R(\theta_1 + \theta_2)$$

- The composition matrix

$$P' = R(\theta_1 + \theta_2) \cdot P$$

Composite Two-Dimensional Scaling

- For two successive scaling operations in 2-D produces the following composite scaling matrix

$$\begin{bmatrix} s_{2x} & 0 & 0 \\ 0 & s_{2y} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} s_{1x} & 0 & 0 \\ 0 & s_{1y} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} s_{1x} \cdot s_{2x} & 0 & 0 \\ 0 & s_{1y} \cdot s_{2y} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{S}(s_{2x}, s_{2y}) \cdot \mathbf{S}(s_{1x}, s_{1y}) = \mathbf{S}(s_{1x} \cdot s_{2x}, \; s_{1y} \cdot s_{2y})$$

General Two-Dimensional Pivot-Point Rotation

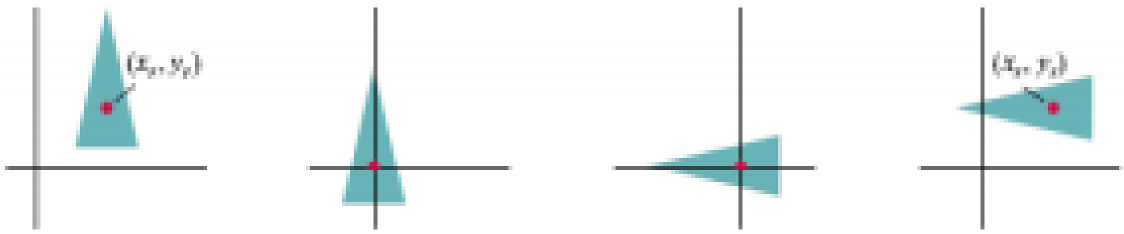
- Graphics package provides only a rotate function with respect to the coordinate origin
- To generate a 2-D rotation about any other pivot point (x_r, y_r), follows the sequence of translate-rotate-translate operations
  1. Translate the object so that the pivot-point position is moved to the coordinate origin
  2. Rotate the object about the coordinate origin
  3. Translate the object so that the pivot point is returned to its original position

General Two-Dimensional Pivot-Point Rotation (2)

$$\begin{bmatrix} 1 & 0 & x_r \\ 0 & 1 & y_r \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & -x_r \\ 0 & 1 & -y_r \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & -\sin\theta & x_r(1-\cos\theta) + y_r\sin\theta \\ \sin\theta & \cos\theta & y_r(1-\cos\theta) - x_r\sin\theta \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{T}(x_r, y_r) \cdot \mathbf{R}(\theta) \cdot \mathbf{T}(-x_r, -y_r) = \mathbf{R}(x_r, y_r, \theta)$$



(a) Original Position of Object and Pivot Point

(b) Translation of Object so that Pivot Point (x_r, y_r) is at Origin

(c) Rotation about Origin

(d) Translation of Object so that the Pivot Point is Returned to Position (x_r, y_r)

FIGURE 5-9 A transformation sequence for rotating an object about a specified pivot point using the rotation matrix $\mathbf{R}(\theta)$ of transformation 5-19.

End of Lecture
Good Luck!

See you

in next lecture...